

Exploring Software Approaches for the Design and Simulation of Bending Active Systems

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Abstract

Emerging directly from a masterclass held by the authors at IASS 2017, this paper presents a quantitative and qualitative benchmark comparison between three distinct software environments, namely SOFiSTiK, Kangaroo and Kiwi3d, framed specifically within the context of designing and simulating bending-active structures. The three environments differ significantly not only in their numerical methods and implementation but also in their stages of software development, licensing structure and design intent and so their comparison represents a timely and valuable insight into the status quo for the design of bending-active hybrid structures.

Keywords: bending-active, hybrid, finite elements, IGA, Kangaroo, SOFiSTiK, Kiwi3d

1. Introduction

The three software environments explored in this paper (SOFiSTiK, Kangaroo, Kiwi3d) employ different methods in their implementation of numerical solvers, pre- and post-processing of geometry, element formulations and user interfacing. Our findings reveal how the mechanical accuracy, speed and interactivity can vary between the environments and detail the pros and cons between discrete and NURBS-based finite elements as well as between numerical solvers based on local or global stiffness. The previous decade has seen a surge of academic interest in and construction of bending-active hybrid structures. This has led to the emergence of numerous software environments which address questions on the form-finding and evaluation of bending-active structures. The software environments presented in this paper represent a selection of the most common approaches currently in use. There are however alternative software solutions which are commercially available or currently under development in the field of bending-active structures, which are not considered here.

2. Methods

The following subchapters aim to describe each of the three methods concisely but with sufficient technical detail to highlight their distinct compositions.

2.1 Method 1: SOFiSTiK (discrete elements, global stiffness-based solver)

The Finite Element Method is a general numerical method for the solution of partial differential equations. FEM has become the de facto standard in the field of structural analysis, and it is commonly used as the tool of choice for a wide variety of engineering problems. Similar to other numerical

methods, the Finite Element Method discretises a continuum with a certain number of elements, hence its name. The method approximates a continuous domain into a finite set of elements.

The Finite Element Method is based on a variational formulation of the principle of virtual work. The principle of virtual work states that the sum of work done by external forces applied on a system must be balanced by the sum of internal work in an arbitrary (but small) virtual displacement. The Finite Element Method is a matrix-based approach, and for static and quasi-static problems it uses implicit integration schemes which solve a system of algebraic equations where the unknowns are represented by the displacements. The Finite Element Method is exemplified in the well-known relationship:

$$\mathbf{Kd} = \mathbf{f}$$

where \mathbf{K} represents the stiffness matrix of the system, \mathbf{d} the vector of unknown displacements and \mathbf{f} the vector of external forces. Typically, the stiffness matrix is derived by considerations of virtual work, although other derivations are also possible.

The commercial software SOFiSTiK is a general-purpose Finite Element based program with a strong focus on structural applications. Besides modules for linear static analysis, SOFiSTiK also implements geometrically non-linear solving schemes for the simulation of large deflections. Benchmarks can be found in [1], [2]. Recently, an Active Bending module (ACTB) was implemented which automatically calculates the internal stress state from a curved beam under the assumption that it was initially straight.

2.2 Method 2: Kiwi3d (NURBS-based elements, global stiffness-based solver)

Kiwi3d, a new plugin for Grasshopper, is based on Isogeometric Analysis (IGA) as introduced by Hughes et al [3] in order to directly integrate structural analysis into CAD. IGA is a subgroup of Finite Element Methods. Its special characteristic lies in the usage of Non-Uniform Rational B-Splines (NURBS) as basis functions for the Finite Elements, which are commonly used for the geometry description in CAD. Hence, a complete reparametrization (meshing) of CAD models for analysis is avoided. The degrees of freedom do not lie on the surface of the element but on the control points of the geometry. The refinement can be done at every stage before and during the analysis without changing the geometry. The applied FEM-kernel is Carat++, which is proprietary research from the Chair of Structural Analysis at the Technical University of Munich. It is available in combination with Kiwi3d as a work-in-progress version. The plug-in Kiwi3d generates a text file in order to communicate with the solver. In turn the plug-in reads the results provided in a further text file for the visualization and evaluation. The text-based interface would also allow the linking of other IGA-enhanced solvers with only small effort. However, Carat++ has advanced features for the non-linear simulation of construction stages and form-finding process, including cutting pattern, which is very beneficial in the design of bending-active hybrid structures. Application examples in this context can be found in [4]–[6].

Further advantages of using IGA in the design of these kind of structures can be seen in the independence of the parametrization for boundary conditions such as loads, supports and coupling entities. All CAD features are available e.g. for the derivation of additional structural members. The method-inherent ability to consistently represent the whole sequence of construction stages during the design process, without losing or having to approximate stresses or displacements, enable a detailed evaluation and enhanced design. The continuous basis functions allow a smooth representation of geometry and results. But this implies as well that discontinuous deformations and stresses within the domain cannot exactly be represented.

2.3 Method 3: Kangaroo (discrete elements, local stiffness-based solver)

The dynamic relaxation (DR) method for iteratively solving structural problems was first developed in the 1960s [7] and has since become well established in many fields. In the built environment, DR is commonly associated with stiffness-independent membrane form-finding [8] or with the distribution of architectural grids over curved surfaces [9]. The form of DR used in Kangaroo differs somewhat from the standard force-based approach, where forces acting on each node are summed to calculate accelerations (from which velocities, then positions are updated), instead making use of projections. “Goals” are defined as functions acting on a set of points, which can describe geometric constraints, elastic material elements, applied loads and other energies. Each goal returns a set of target positions for

each point it acts on, projecting them to their closest zero energy state. This can result in multiple target positions for each point, depending on the number of goals acting on it, and these are then combined in a weighted averaging step. The solver alternates between these two simple local steps to minimize the global energy - a form of the alternating direction method of multipliers (ADMM)[10].

Since the strengths of the goals relative to one another are given by the scalar weighting factors in the averaging step, this keeps the new position for each point within the convex hull of its target points, avoiding the problem of ‘overshooting’ which can occur in the force based DR method, where elements with high stiffness can result in large acceleration vectors, leading to instability, and reducing the timestep sufficiently to mitigate this can be difficult or lead to impractically slow convergence. Another simple projection-based constraint satisfaction technique which has been widely used in games and animation is to apply local constraints sequentially [11], iterating over them with a Gauss-Seidel method. However, this is not suitable where quantitatively correct physical deformations are sought, since the ordering of the constraints and number of iterations performed all affect the result, making it difficult or impossible to tune for real material stiffness parameters. By contrast Kangaroo performs all the projections for one step independently and in parallel, and the energy minimised is exactly the sum of the squared distances of the points from the target positions of all the goals, each multiplied by the weighting factor for that goal. This allows for quantitative simulation of deformations with real material stiffness parameters.

Finite elements can be used here as goals, the key difference with the more common form of FEM being that the global stiffness matrix does not need to be computed, since only local stiffness is used. This technique is also closely related to that described in [12], but there the step in which the projections are combined involves a global solve. By contrast both steps in the Kangaroo solver are local, meaning topology can be freely modified during simulation, and making the definition of new goals simple and flexible, since they need only return a target position and scalar weighting. Furthermore, in Kangaroo the projection method is combined with a modified version of the ‘drift damping’ described in [13] technique to accelerate convergence more effectively than the usual kinetic or viscous damping.

Generally, a prerequisite for implicitly integrated methods more common with FEM is that the systems must be statically determinate or indeterminate. Mechanisms can cause numerical instability and are more difficult to solve. Kangaroo however is insensitive to the static determinacy of a structural system such that mechanisms and large deformations are not an issue, provided the solver is able to remain stable (as is usually the case). This insensitivity to static determinacy and large deformations is highly suited to experimental and interactive design environments.

3. Benchmarks

While the subject matter presented in this paper merits in-depth and comprehensive technical benchmarking covering many aspects relating to software and engineering performance, this paper presents one quantitative and one qualitative benchmark for a concise overview.

3.1 Inextensible Cantilever

Large deformations feature heavily in bending-active hybrid structures. Therefore, a classical benchmark for very large deformations is chosen for the comparison of the three methods with an analytical solution derived by Mattiasson [14]. A cantilever with length $L=10$, bending stiffness $EI=100$ and axial stiffness $EA=1e9$ is loaded by increasing load increments with a maximum value $F_{max} = 10$ at the tip as shown in Figure 1. For each software environment several discretization resolutions, i.e. number of elements n , are compared and plotted against the analytical solution in Figure 2. Note that for IGA the term ‘element’ corresponds to a non-zero knot span of the underlying NURBS geometry.

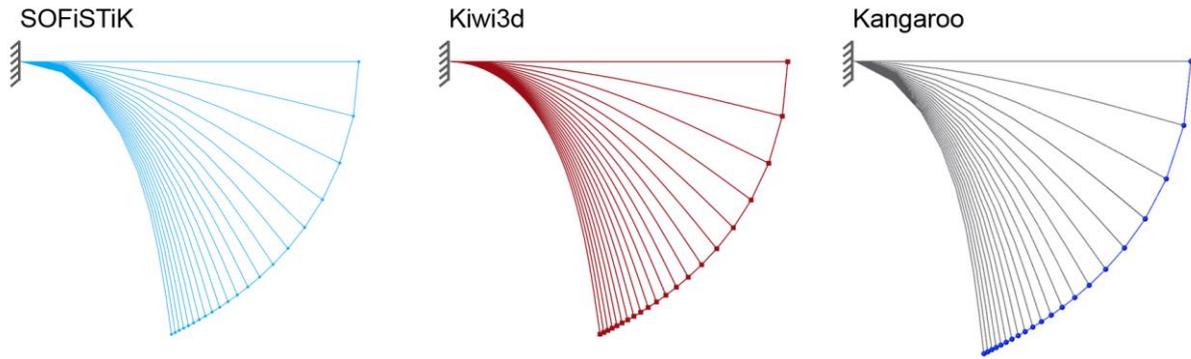


Figure 1: Deformed cantilever under tip loading with 8 discretized elements for all 20 load steps.

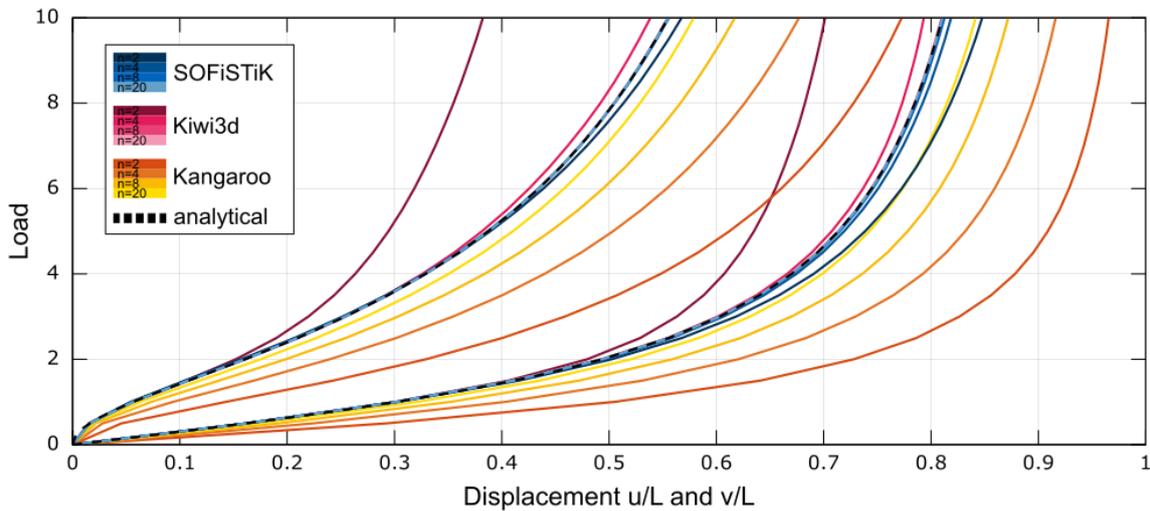


Figure 2: Relative displacement u/L and v/L of the cantilever's tip computed with SOFiSTiK, Kiwi3d and Kangaroo for different numbers of elements n and load steps.

This benchmark test aims to evaluate exclusively the accuracy of method's bending stiffness with an effectively inextensible beam (extremely high value for axial stiffness EA). The results are valuable, particularly within the context of bending-active structures. However, it should be noted that the Finite Element formulations from each method are not the same. Kangaroo's bending formulation is based on Barnes' simplified model of three sequential vertices with 3 DOF [15]. SOFiSTiK makes use of Timoshenko's beam theory and IGA uses Euler Bernoulli beam. As such the results from this benchmark represent only one aspect (accuracy) within the context of designing and building bending-active structures.

As is to be expected, the simulations with the lowest number of discretized elements return the largest errors. For the u/L deflection, IGA shows a deviation of -13.5% from the analytical model and Kangaroo features a deviation of 19.1% whereas SOFiSTiK features only 4.6% error. At a discretisation of 20 elements, all methods feature high precision with errors for v/L deflection at -0.001% for IGA, -0.003% for SOFiSTiK and 4.305% for Kangaroo. It may be noted, that in real terms 4.305% error for a 10m cantilever equates to 24mm.

3.2 Complex Shape: Elastica Tetrahedron

A key objective for bending-active structures is to achieve high stiffness and low weight from very slender beams. This usually involves forming and then constraining the bending-active elements such that they occupy a large volume which, if executed well, will provide the high stiffness and low weight

sought after. The second benchmark presented here was performed as part of the 2017 IASS workshop and comprises two slender beams which are bent in multiple axes and fixed to one another at three locations approximating a tetrahedron. Such a system may be stabilised and stiffened by the addition of tensile membrane or cable elements. To achieve this complex shape, each of the three methods adopt substantially different processes (described below) with distinct advantages and disadvantages relating to accuracy, speed, user interfacing and design interactivity.

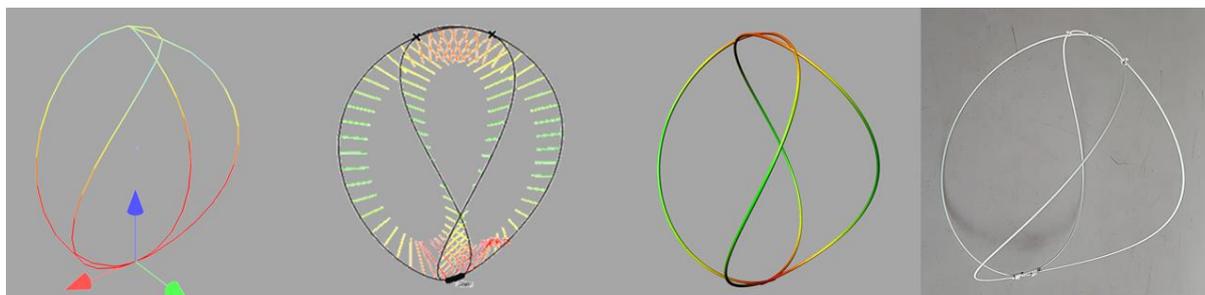


Figure 3: Elastica tetrahedron: complex 3D form from slender and actively-bent GFRP rod. From left to right: SOFiSTiK, Kangaroo, Kiwi3d and mock-up.

3.2.1 SOFiSTiK

The most common approach in SOFiSTiK for the simulation of bending-active structures is by using contracting cables [16]. The cables are given an arbitrary and low axial stiffness so that under a prestress load they contract rapidly by transferring this prestress load directly to the ends of and aligned to the cable. The rate of contraction can be defined by the user to follow a polynomial relationship per iteration or it can be linked to load results from the cable from the previous iteration step. In this way the rate of contraction adapts to accommodate the nonlinearities of the form finding process. As the deformation path of the elements is dependent on intermediate stages of changing equilibrium, contracting cables have the advantage of reducing inelegant or artificial restraining forces during the form-finding process. This is achieved by making the structure follow the deformation path that it would naturally occur from the gradual contraction of the cables.

In the case of the tetrahedron, the structural system is modelled with the same dimensions, units, material properties and cross-section values as in reality. This is necessary when simulating bending-active systems with SOFiSTiK, as the software tends to be sensitive to non-realistic and off the charts values. The simulation process follows in two stages. In the first step the straight rod is iteratively bent into a loop until the end points meet. For the second step a transversal contracting cable is added to bend the loop out of the plane and achieve the spatial tetrahedron. For the second step, a small imperfection in the form of a vertical force is needed to induce the initial buckling of the system. For the import/export of CAD geometry between Rhino and SOFiSTiK the official SofRhino plugin by SOFiSTiK as well as the Rhino/Grasshopper interface STiKbug were used [17].

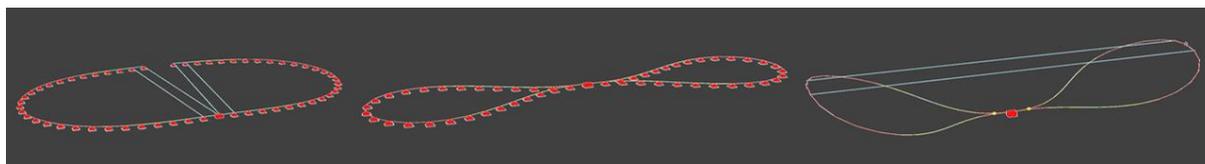


Figure 4: Different stages of the simulation process for the elastica tetrahedron in SOFiSTiK. From left to right: The rods are bent using a series of contracting cables, the loops are formed and secured together with coupling elements and finally a new series of contracting cables pull the rods together.

3.2.2 Kiwi3d

Contracting cables are also used in Kiwi3d in order to model the elastica tetrahedron similar to SOFiSTiK. In a first step the beam is bent to a loop with two cables for aligning the ends. In the second

step the result can be mirrored and be used as initial geometry, which is still aware of the first bending process. Forces are used in the first load step to push the beams out of the ground plane. Two cables connect the upper part of the loops. Note that the lower parts are not blocked against torsion as a torsional connecting of the real GFRP model was neither possible. A torsional support would slightly modify the result. Alternatively, the automatically derived pre-stress (similar to SOFiSTiK's ACTB module) can be used to directly model the structure if torsional moments are not expected. The whole analysis data is available and can be evaluated in the CAD environment. Consequently, also other functions are callable and e.g. an optimization w.r.t. the final structure gets possible by changing the geometry and analysis parameters simultaneously.

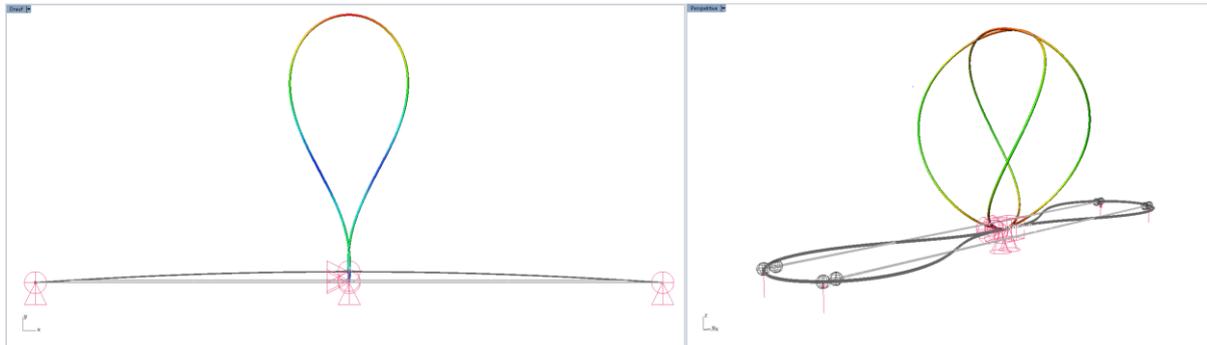


Figure 5: Design process of the elastica tetrahedron within Kiwi3d: Left: rod is bent by two cables in its first construction stage to a loop. Right: two clamped loops are connected and pulled together by cables in order to derive its final shape.

3.2.3 Kangaroo

A custom computational pipeline [18] developed, primarily by Anders Holden Deleuran, for the SmartGeometry conference in 2016 was introduced at the IASS 2017 workshop and was used in order to generate the tetrahedron benchmark geometry with Kangaroo. The computational pipeline harnesses Rhino and Grasshopper's many geometric strengths (via the RhinoCommon library) to allow users to design polygonal topology assemblies (comprised of beams, cables, intersections thereof and supports). The pipeline discretises the geometry, assigns topology logic, part indexing, material properties and then sends this data to Kangaroo to solve, on the fly. Since the target angle between sequential polyline segments in the Kangaroo goal for bending stiffness can be freely defined, both linear and polygonal initial geometries which result in curved elastica can be solved. This novel approach allows users to create a design, explore topological variations and modify accordingly based on real-time feedback on structural performance. It also meant that the complex form in this benchmark could be sketched and solved within a matter of minutes.

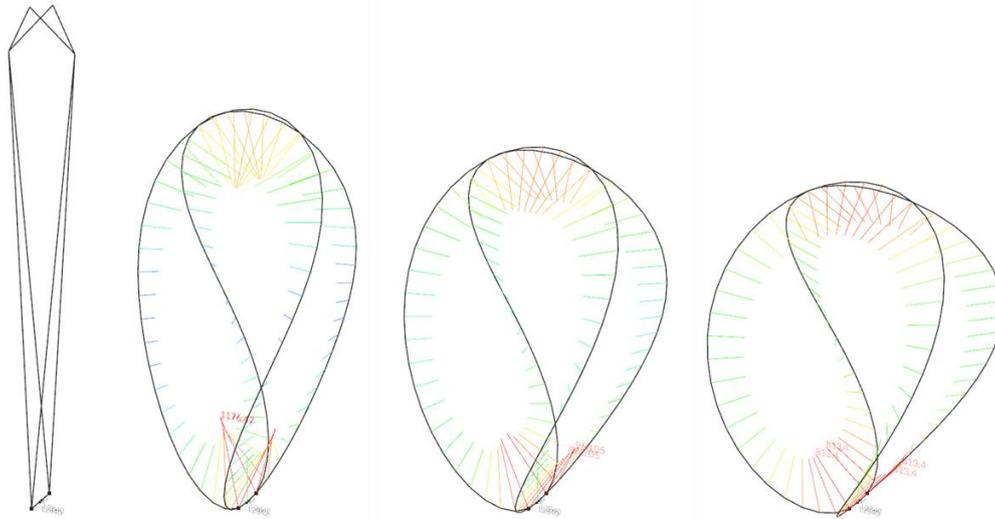


Figure 6: Custom computational pipeline for Rhino / GH in which the user draws a polygonal topology representation of a design (left) which is solved for equilibrium on the fly by Kangaroo (right) allowing for rapid design exploration & evaluation.

4. Conclusion

This paper offers a timely insight into one established and two emerging software environments each capable of modelling, shaping and simulating bending-active structures: a feat that not many FE packages can achieve due to their large deformations, complex geometries and geometrically non-linear effects. The first benchmark compares deformations from an inextensible tip-loaded cantilever and reveals how SOFiSTiK is rightly celebrated as one of the industry's most precise and dependable FE packages. The second benchmark models a geometrically complex elastica tetrahedron and reveals much about the, often overlooked, aspects of topology generation and user interfacing. Despite improvements from plugins, SOFiSTiK's FORTRAN and CAD-based data input methods are showing their age in a world of parametric modelling. The Carat++ solver behind Kiwi3d is reaffirmed as highly robust and precise while the bold new world of isogeometric analysis offers a tantalizing leap of progress for FEM in seamlessly combining NURBS modelling with physical simulations by eluding (or at least greatly simplifying) discretisation. Kangaroo offers users an unparalleled freedom to script geometric constraints or FE formulations to suit their own requirements. The stability of Kangaroo's novel projection-based dynamic relaxation solver introduces a completely fresh level of interactivity, accessibility and customizability to the world of Finite Element Modelling.

Each of the three methods discussed in this paper offer unique and significant strengths with respect to the modelling of bending-active structures.

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